Eddy-Current-Effect Homogenization of Windings in Harmonic-Balance Finite Element Models Coupled to Nonlinear Circuits

Ruth V. Sabariego[1], Korawich Niyomsatian [1,2], and Johan Gyselinck [2]
[1] KU Leuven, Dept. Electrical Engineering, Campus EnergyVille, Belgium, ruth.sabariego@kuleuven.be
[2] Université Libre de Bruxelles (ULB), BEAMS Dept., Belgium, johan.gyselinck@ulb.ac.be

This paper deals with the harmonic-balance finite element analysis of a multi-turn winding device coupled to electrical circuits comprising nonlinear components. The eddy-current effects in the windings are accounted for via a frequency-dependent reluctivity and impedance with a homogenization technique. The proposed multi-frequency approach is validated through a single-phase four-diode rectifier with an axisymmetric FE model of an inductor. The harmonic-balance and time-stepping results are compared.

***Index Terms*—Eddy currents, finite element methods, harmonic balance, proximity effect, skin effect, nonlinear control systems.**

I. Introduction

Accounting for the sometimes non-negligible eddy-current effects in the windings of electromagnetic devices requires a brute-force finite element (FE) model with a fine discretization of each turn, which is too computationally expensive. Frequency-domain homogenization techniques provide a closed-form continuous representation of the homogenized winding [1]. In [2], frequency-dependent proximity and skin effect parameters are identified in a general approach and straightforwardly included in a FE model. This flexible winding homogenization technique is further embedded in a harmonic-balance (HB) FE approach in [3].

In this paper, we aim at studying a homogenized multi-turn winding coupled to an electrical circuit with nonlinear components via a HB-FE technique, which may be an efficient alternative to plain time-stepping in case of long transients [4]. Among the available implementations, e.g. [5], we adopt the Galerkin time-domain variant in [6]. The homogenized field-nonlinear circuit coupled model is validated by means of a single-phase four-diode rectifier and an inductor (Fig. 1).

II. HB-FE Models with nonlinear circuits

Let us consider an electrical circuit that comprises a number of conductors in the FE domain (with e.g. a magnetic vector potential formulation) and a number of lumped resistances, inductances and voltage sources We introduce loop currents linked to a set of independent oriented current loops in the circuit. We can write the following generic nonlinear system:

$$\mathbf{M}(\mathbf{X}(t))\,\mathbf{X}(t)+\mathbf{N}(\mathbf{X}(t))\,\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}t}=\mathbf{F}(t)\,, \tag{1}$$

where $\mathbf{X}(t)$ is the column vector of unknowns; $\mathbf{M}$ is the matrix comprising time-independent FE blocks, connectivity matrices and square loop resistance matrices; $\mathbf{N}$ is the matrix accounting for the time-dependent FE blocks and square loop inductance matrices; and $\mathbf{F}(t)$ includes the column vectors with the sources. The nonlinearity in $\mathbf{M}$ may be due to field-dependent reluctivities (e.g. reversible material law) or to current-dependent resistances in the system (e.g. the on and off resistances of a diode). A dynamic current-dependent lumped component can be included in matrix $\mathbf{N}$ (e.g a shelf).

The HB approach allows obtaining the periodic steady-state solution by solving a unique, but larger and denser, system of algebraic equations. The n coefficients in $\mathbf{X}(t)$ can be expressed as a truncated Fourier series considering n_f frequencies f_k, $k=1,2,\ldots$, integer multiples of the fundamental frequency f_1 (period $T=1/f_1$), or pulsations $\omega_k=2\pi f_k$, for a total of $n_\mathrm{h}=2n_\mathrm{f}$ harmonic (cosine or sine) basis functions (BFs), and $n\cdot n_\mathrm{h}$ unknown coefficients collected in column vectors $\boldsymbol{\mathcal{H}}(t)$ and $\mathbf{X}^\mathrm{H}$ respectively:

$$\boldsymbol{\mathcal{H}}(t)=\left[h_1\cdots h_{n_\mathrm{h}}\right]^\top=\left[\cdots\ \cos(\omega_k t)\ \ \sin(\omega_k t)\ \ \cdots\right]^\top, \tag{2}$$

$$\mathbf{X}(t)=\left(\mathbb{1}\otimes\boldsymbol{\mathcal{H}}(t)^\top\right)\,\mathbf{X}^\mathrm{H}\,, \tag{3}$$

where $\otimes$ denotes the Kronecker product. For the sake of simplicity, a dc term (with unitary BF) is not considered [6].

These n_h cosine and sine BFs are mutually orthogonal and coupled via their time derivative:

$$\frac{2}{T}\int_0^T\boldsymbol{\mathcal{H}}(t)\,\boldsymbol{\mathcal{H}}^\top(t)\,\mathrm{d}t\;=\;\mathbb{1}\,, \tag{4}$$

$$\boldsymbol{\mathcal{Q}}=\frac{2}{T_1}\int_0^T\boldsymbol{\mathcal{H}}(t)\,\frac{\mathrm{d}\boldsymbol{\mathcal{H}}^\top}{\mathrm{d}t}\,\mathrm{d}t=\begin{bmatrix}\cdots&\cdots&\cdots&\cdots\\\cdots&0&\omega_k&\cdots\\\cdots&-\omega_k&0&\cdots\\\cdots&\cdots&\cdots&\cdots\end{bmatrix}. \tag{5}$$

The ODEs (1) are weakly imposed using the $h_j(t)$ BFs [6]:

$$\frac{2}{T}\int_0^T\left(\mathbf{M}\,\mathbf{X}+\mathbf{N}\,\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}t}\right)h_j\,\mathrm{d}t=\frac{2}{T}\int_0^T\mathbf{F}\,h_j\,\mathrm{d}t\,, \tag{6}$$

leading to one system of $n\cdot n_\mathrm{h}$ algebraic equations, that in the linear case reads:

$$\mathbf{M}^\mathrm{H}\mathbf{X}^\mathrm{H}\;=\;\mathbf{F}^\mathrm{H}\,,\quad\mathbf{M}^\mathrm{H}=\mathbb{1}\otimes\mathbf{M}+\boldsymbol{\mathcal{Q}}\otimes\mathbf{N}\,, \tag{7}$$

$$\mathbf{F}^\mathrm{H}=\frac{2}{T}\int_0^T\boldsymbol{\mathcal{H}}(t)\otimes\mathbf{F}(t)\,\mathrm{d}t\,. \tag{8}$$

In case of nonlinearity, the Newton-Raphson (NR) method is a good choice for linearizing the nonlinear HB system (6). At the material level, the harmonic differential reluctivity tensor depends on the differential reluctivity tensor which in turn depends on the harmonic component of the induction field. Analogously, the current-dependent lumped resistances (e.g. diodes) are characterized by harmonic differential resistances that depend on the variation of differential resistances [6]. The time integration in (6) is performed numerically considering a sufficiently large number of equidistant and equal-weighted time instants in $[0, T]$.

III. Homogenization of Windings

In (1) eddy currents are explicitly accounted for via a conductivity-dependent eddy-current matrix (classical FE) or via a frequency-dependent complex reluctivity [2] in the homogenized winding window (one stranded inductor) and a complex impedance replacing the dc resistance.

In the multi-harmonic case, we adopt a different proximity-effect complex reluctivity $\boldsymbol{\nu}_{prox}(f_k)$ and skin-effect impedance $\boldsymbol{Z}_{skin}(f_k)$ per considered frequency in the HB approach [3]. The cosine and sine HB-BFs are coupled due to these effects. The matrix $\mathbf{M}^{\mathrm{H}}$ is modified by including $\boldsymbol{\nu}_{prox}(f_k)$ in the reluctivity-dependent stiffness matrix $\mathbf{S}(\nu)$ as:

$$\begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ \cdots & \mathbf{S}(\Re(\boldsymbol{\nu}_{prox}(f_k))) & \mathbf{S}(\Im(\boldsymbol{\nu}_{prox}(f_k))) & \cdots \\ \cdots & -\mathbf{S}(\Im(\boldsymbol{\nu}_{prox}(f_k))) & \mathbf{S}(\Re(\boldsymbol{\nu}_{prox}(f_k))) & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}, \tag{9}$$

and by integrating $\boldsymbol{Z}_{prox}(f_k)$ in the circuit coupling blocks linking currents and voltages in stranded conductors:

$$\begin{bmatrix} \cdots & \cdots & \cdots & \cdots \\ \cdots & \Re(\boldsymbol{Z}_{skin}(f_k)) & \Im(\boldsymbol{Z}_{skin}(f_k)) & \cdots \\ \cdots & -\Im(\boldsymbol{Z}_{skin}(f_k)) & \Re(\boldsymbol{Z}_{skin}(f_k)) & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix}. \tag{10}$$

IV. Application example

By way of validation, we consider a single-phase four-diode rectifier with a sinusoidal voltage supply (50 V peak, $f = 10\,$kHz), a smoothing inductor, a smoothing capacitor ($C = 10\,\mu$F) and a load resistance $R_{\mathrm{dc}} = 100\,\Omega$. The diode resistances are $R_{\mathrm{on}} = 10^{-1}\,\Omega$ and $R_{\mathrm{off}} = 10^{5}\,\Omega$. The inductor is the multi-turn coil described in [2]. It is modelled by FEs accurately accounting for eddy-current effects.

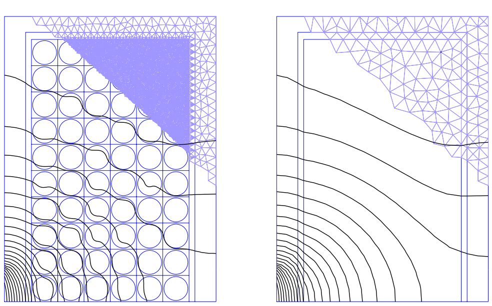

Fig. 1. Flux lines in winding domain obtained with the fine (left) and homogenized model (right) at $f = 50\,$kHz. Detail of the meshes.

Time-stepping simulations (no homogenization) are carried out for validating the HB approach (with homogenization). The long transient (150 periods) is time-stepped with $\Delta t = T/240$. The dc-term and all harmonics are present in the currents at the ac-side of the rectifier and at the inductor (dc-side). The corresponding current waveforms computed with the homogenized winding HB approach are compared to the reference TD result in Fig. 2. HB calculations are carried out considering first only the dc-term and the fundamental frequency (denoted by HB 2), and then gradually expanding the spectrum with harmonics, up to the 99th harmonic (HB 100). One observes that from HB 50 on, the agreement with the time-stepping results is excellent at the coil level. More harmonics are needed at the ac-side due to the sharp switching of the diodes.

The NR process converges well, even in the presence of the diodes in the electrical circuit. If the NR process is initialised with the previous HB solution (with less frequencies), the computation time is considerably reduced.

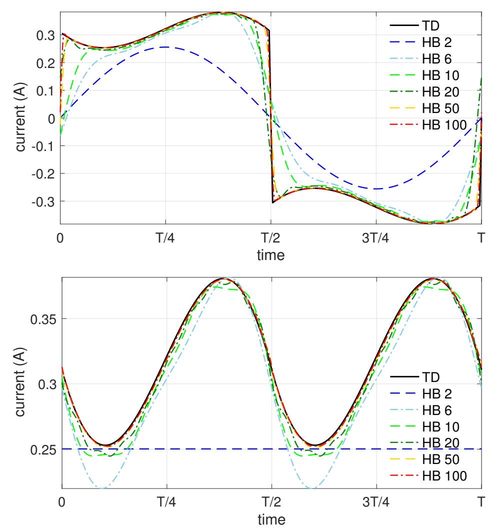

Fig. 2. Current waveforms at the ac-side of the rectifier (up) and at the coil, dc-side (down).

Further results and a thorough discussion on the computational cost will be included in the full paper.

Acknowledgment

Work supported by the Belgian Science Policy (IAP P7/02) and the Walloon Region under WBGreen-FEDO grant (convention n. 1217703).

References

[1] D. C. Meeker, "An improved continuum skin and proximity effect model for hexagonally packed wires," *J. Comput. Appl. Math.*, vol. 236, no. 18, pp. 4635-4644, 2012.

[2] J. Gyselinck, R. V. Sabariego and P. Dular, "Time-domain homogenization of windings in 3-D finite element models," *IEEE Trans. Magn.*, vol. 44, no. 6, pp. 1302-1305, 2008.

[3] R. V. Sabariego, J. Gyselinck, "Eddy-Current-Effect Homogenization of Windings in Harmonic-Balance Finite Element Models," In Proceedings of CEFC 2016, Miami, Florida, USA, November 1316, 2016.

[4] S. Yamada and K. Bessho, "Harmonic field calculation by the combination of finite element analysis and harmonic balance method," *IEEE Trans. Magn.*, vol. 24, no. 6, pp. 2588-2590, 1988.

[5] S. Ausserhofer, O. Biro, and K. Preis, "An efficient harmonic balance method for nonlinear eddy-current problems," *IEEE Trans. Magn.*, vol. 43, no. 4, pp. 1229-1232, 2007.

[6] J. Gyselinck, P. Dular, C. Geuzaine, W. Legros, "2D harmonic balance FE modelling of electromagnetic devices coupled to nonlinear circuits," *COMPEL*, vol. 23, no. 3, pp. 800-812, 2004.